Experiment MP-1

e/m ratio of the electron

Educational Objective

To study the charge-to-mass (e/m) ratio of cathode rays (electrons) in a cathode-ray tube (CRT).

Experimental Objective

To measure the deflection of an electron beam in a crossed electric and magnetic field and determine the charge-to-mass ratio.

Apparatus

One Tel-Atomic CRT tube containing:

1. an "electron gun,"
2. a grid to measure the electron beam deflection, and
3. an electric field grid to deflect the electrons.
4. One pair of Helmholtz coils to produce a homogeneous magnetic field.

Voltage and current meters to:

1. monitor the potentials on the electron gun and the deflection grid, and
2. monitor the current in the magnetic field (Helmholtz coils).

Method

First, accelerate the electrons through the "electron gun." Second, deflect them using a constant electric field. Third, adjust the current in the magnet (Helmholtz coils) to produce a zero deflection of the electron beam. The magnetic field vector, **B**, must be perpendicular to both **v** (the velocity of the electrons), and **E** (the electric field).

Using the measurements of the electric and magnetic fields, along with the geometrical measurements of the trajectory, the e/m ratio of the electron can be calculated.

Theory

If the electron is a particle of finite charge e and has mass m it will obey the laws of motion.

In the TEL 525 tube the electrons are accelerated through the electron gun by traversing a potential difference, V. The electrons emerge from the electron gun with an increased kinetic energy arising from conservation of energy:

$$\frac{1}{2}mv^2 = qV \tag{1}$$

where:

- v is the velocity in the x direction,
- q is the charge of the electron, and
- V is the potential difference.

If the velocity of the electrons emerging from the gun is known, the e/m ratio can be calculated using the above equation (1) where $q = e$.

The electric field in the $-y$ direction will deflect the electrons upward so that a deflection in the y direction as a function of x (or L in this case) can be observed. The deflection in y direction can be calculated using Newton's 2nd Law, eE = ma. Solving for the acceleration in the $+y$ direction, we find that

$$a_y = \frac{e}{m}E_y .$$

The deflection in the y direction can be calculated from

$$y = v_{oy}t + \tfrac{1}{2}at^2 ,$$

the displacement due to constant acceleration.

Substituting the acceleration a_y and v_{oy} into this equation, we obtain

$$y = \frac{eEL^2}{2mv_x^2} \qquad (2)$$

where

- y is the deflection of the electron beam,
- e is the electric charge (1.602×10^{-19} coulombs),
- E is the electric field (volts/meter),
- L is the horizontal distance traversed across the vertical electric field,
- v_x is the velocity of the electrons emerging from the electron gun, and
- e/m is the charge-to-mass ratio.

Aside from the charge-to-mass ratio (e/m), the velocity of the electron, v_x, is the only unknown in the deflection equation (2). In order to measure v_x we cross the *vertical electric field* with a *horizontal magnetic field* in order to produce a beam with zero deflection. The deflection due to the vertical electric field is canceled by the deflection due to the horizontal magnetic field. Since the sum of the external forces is zero during "no deflection," the electric force must be equal to the magnetic force (Lorentz force).

$$e\vec{E} = e\vec{v} \times \vec{B} \qquad (3)$$

From this equation we find that

$$v_x = \frac{E}{B} \qquad (4)$$

where:

- E is the deflecting electric field (volts/meter), and
- B is the magnetic field (tesla) required to cancel the deflection.

Substituting equation (4) into equation (2) we find that the deflection is:

$$y = \frac{e}{m}\frac{B^2L^2}{2E} \quad . \qquad (5)$$

Using equation (5) we can now determine the *e/m* ratio.

Procedure

1. Turn on the ammeters.
2. Turn the coarse and fine knobs on the high voltage supplies (5000V and 2000V) completely counter-clockwise to make sure the voltage is at a minimum before turning them on. The "fine" adjustment knobs may be broken on some of the supplies. You don't need to use them.
3. Turn on the two high voltage power supplies. **Note: one of the high voltage supplies provides voltage to both the electron gun (HV) and the filament (6VAC), (i.e., the incandescent lamp supplying the source of electrons).**
4. Adjust the potential for the "electron gun" using the *coarse* knob until the voltage reads between 2000 and 3000 volts.
5. Next adjust the potential for the "deflection grid" using the coarse knob until the deflection is near the maximum at (approximately 2.5 cm in the y-direction for 10 cm in the x-direction).
6. Record the electron gun voltage: _________________ volts.
7. Record the grid voltage: __________ volts *(V)* and its uncertainty _____________ volts *(δV)*.
8. Calculate the electric field (*E*): $E = V/d$ __________________ V/m and its uncertainty _______________ V/m. *(See the section on Error Analysis)*
9. Record the deflection *(y)*: ___________ m and its uncertainty _________________ m.

   Record the length *(L)*: ______________ m and its uncertainty _________________ m.
10. Adjust the current in the Helmholtz coils until zero deflection is obtained. Read the current off the ammeter.
11. Record the current in the magnet: _____________ A and its uncertainty ________________ A.

In order to calculate the magnetic field, use the following equation for the B field in a Helmholtz coil:

$$B = \frac{8n}{5\sqrt{5}}\frac{I\mu_o}{a} \quad \text{[tesla]} \qquad (6)$$

where

- n = no. of turns = 320 turns
- I = the current (amperes)
- a = mean radius (0.068 m)
- $\mu_o = 4\pi \times 10^{-7}$ webers/(ampere x meter)
- B = magnetic field (tesla)

Note: 1 tesla = 1 weber / meter^2.

12. Calculate the magnetic field *(B)*.

 _____________ tesla.

 Calculate the uncertainty in the magnetic field _____________ tesla.

13. Calculate the velocity of the electrons using $\upsilon_x = E/B$.

 ________________ m/s.

14. Calculate the e/m ratio.

 ________________ C/kg.

15. Calculate the uncertainty in e/m:

 ________________ C/kg.

Question #1

How does the calculated e/m ratio from this experiment compare to the known value of e/m?

Question #2

How does the strength of the magnetic field due to the Helmholtz coils compare to the local magnetic field due to the earth?

Question #3

What is your final answer for the charge-to-mass ratio? You must express it in the following form:

$$e/m = (e/m)_{calculated} \pm \delta(e/m)$$

See the Error Analysis section for more details.

Question #4

What would you recommend to improve the e/m measurement you've just made?

Error Analysis

After recording your measurements and their respective uncertainties, it is now possible to calculate the uncertainty in the final measurement. As an example, let's calculate the relative error of the electric field between the deflection plates. The electric field is calculated from the equation:

$$E = \frac{V}{d}$$

where V is the potential (volts) and d is the separation between the plates (meters). The relative uncertainty in the electric field can be calculated using the following:

$$\frac{\delta E}{E} = \sqrt{\left(\frac{\delta V}{V}\right)^2 + \left(\frac{\delta d}{d}\right)^2}$$

where the relative uncertainties of the *potential* and the *separation* are added in quadrature.

When quoting your answer for e/m (the charge-to-mass ratio), you must quote both the mean value and the uncertainty. The equation for calculating the mean value of e/m is:

$$\frac{e}{m} = \frac{2Ey}{B^2L^2} \qquad (7)$$

where B is the magnetic field (tesla), E is the electric field (volts/meter), y is the displacement (meters), and L is the length of the deflection grid (meters).

To determine the uncertainty in e/m, you must first calculate the relative uncertainty for e/m.

$$\frac{\delta(e/m)}{e/m} = \sqrt{\left(\frac{\delta y}{y}\right)^2 + \left(\frac{\delta E}{E}\right)^2 + \left(\frac{2\,\delta B}{B}\right)^2 + \left(\frac{2\,\delta L}{L}\right)^2}$$

where the relative uncertainties of the measured quantities are added in quadrature.

The final answer should be quoted as:

$$e/m = (e/m)_{calculated} \pm \delta(e/m).$$

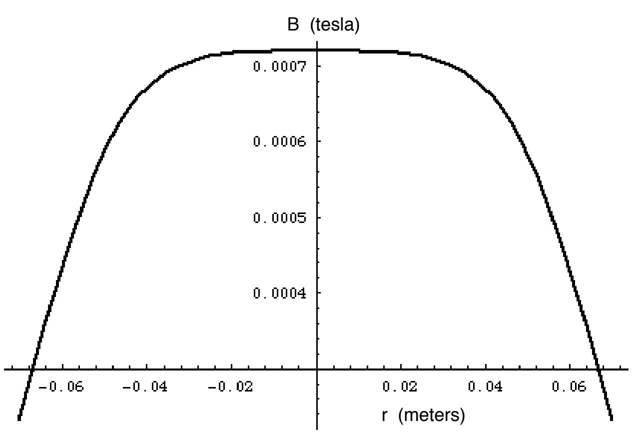
The Magnetic Field

The magnetic field for <u>one</u> of the two Helmholtz coils can be written using the Biot-Savart Law:

$$B_z = nI \times 10^{-7} \int_0^{2\pi} \frac{a(a - r\cos\theta)\ d\theta}{\left(a^2 + b^2 + r^2 - 2ar\cos\theta\right)^{3/2}} \qquad (8)$$

where n is the number of turns, a is the radius of the coil (meters), b is the distance away from the plane of the coil (meters), and r is the distance from the axis of symmetry (perpendicular to the plane of the coil and passing through the center of the coil).

All Helmholtz coils are oriented such that the separation between the two coils is a (the radius of an individual coil). At this separation, the magnetic field in the midplane between the two coils (i.e., at b = a/2) is very homogeneous. This is shown in the figure below. However, as one moves further away from the axis of symmetry (r=0) and approaches the radius of the Helmholtz coil (r $\rightarrow$ 0.068 meters), the magnetic field is observed to decrease rapidly.



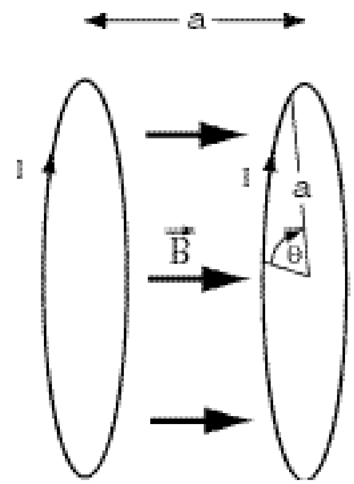
Magnetic Field as a function of r

The above figure shows the magnetic field (B in tesla) in the mid-plane between the two coils versus r (the distance from the axis of symmetry) for n = 320 turns, i = 170 mA, and b = a/2, where a = 0.068 meters.

If you take the above equation for B_z (eq. 8) and set b = a/2 and r = 0, integrate it, and multiply by two (because there are two coils) you will get the equation

$$B = \frac{32\pi n}{5\sqrt{5}} \frac{I}{a} \times 10^{-7} \ (tesla)$$

as seen above (eq. 6).



Helmholtz Coils (side view)